

# The nuclear AC-Stark shift in super-intense laser fields

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The direct interaction of super-intense laser fields in the optical frequency domain with nuclei is studied. As main observable, we consider the nuclear AC-Stark shift of low-lying nuclear states due to the off-resonant excitation by the laser field. We include the case of accelerated nuclei to be able to control the frequency and the intensity of the laser field in the nuclear rest frame over a wide range of parameters. We find that AC-Stark shifts of the same order as in typical quantum optical systems relative to the respective transition frequencies are feasible with state-of-the-art or near-future laser field intensities and moderate acceleration of the target nuclei. Along with this shift, we find laser-induced modifications to the proton root-mean-square radii and to the proton density distribution. We thus expect direct laser-nucleus interaction to become of relevance together with other super-intense light-matter interaction processes such as pair creation.

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## I. INTRODUCTION

In most branches of physics, a controlled manipulation of the considered system has proven to be extremely useful to study fundamental system properties, and to facilitate a broad range of applications. A prominent example for this is quantum optics or laser physics in general [1, 2, 3], for instance related to light-matter interactions on the level of single quantum objects [4, 5]. Similar control is also possible at lower driving field frequencies, e.g., with NMR techniques in the microwave frequency region [6]. Towards higher frequencies, in particular the development and deployment of high-intensity lasers have opened the doors to new fascinating areas of physics of light-matter interactions. Laser fields reach and succeed the Coulomb field strength experienced by the electrons due to the nucleus and thus give rise to a plethora of exciting phenomena [7, 8, 9].

The above examples have in common that they focus on the interaction of the driving fields with the outer electron shell of the atoms. Regarding the interaction of strong laser fields with nuclei, however, mostly *indirect* reactions have been studied so far. In these reactions, electrons or plasmas are encountered by a laser pulse and then, directly or by creating radiation, react with the nucleus. Typical examples are the production of MeV X-rays in a plasma that is generated by femtosecond laser pulses [10], the study of  $\gamma$ -induced nuclear reactions in plasma radiated by a super-intense laser [11, 12],

or neutron production in laser plasma [13, 14]. Also, the coupling of nuclear and electronic transitions has been considered [15]. Applications are lasing [16], the control of Mössbauer spectra [17, 18], or inversionless amplification [19]. Further applications include optically induced nuclear fission [20] and fusion [21, 22], nuclear reactions, isomer excitations [23], or nuclear collisions [24].

In contrast, *direct* laser-nucleus interactions do not involve mediating particles such as electrons or gamma ray photons. So far, however, such direct interactions have mostly been dismissed because of small interaction matrix elements [25]. Rare exceptions study direct laser- and x-ray-nucleus interactions in the context of  $\beta$  decay [26] or x-ray-driven gamma emission of nuclei [27]. On the other hand, effects such as laser-induced pair creation which previously had been neglected for the same reason of small interaction matrix elements, are now being studied, see for example [28, 29, 30, 31, 32, 33, 34]. The reason is that it can be expected that present and upcoming technology will allow to enter regimes where these traditionally neglected processes become possible.

In [35], we have shown that direct laser-nucleus interactions may indeed become of relevance in future experiments employing x-ray lasers, opening the field of nuclear quantum optics. In particular, the coherence of the laser light expected from new sources such as TESLA XFEL [36] is the essential feature which may allow to access extended coherence or interference phenomena reminiscent of atomic quantum optics. Such laser facilities, especially in conjunction with moderate acceleration of the target nuclei to match photon and transition frequency, may thus enable one to achieve nuclear Rabi oscillations, photon echoes or more advanced quantum optical schemes [1] in nuclei.

This in principle may allow for a considerable range of

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applications: As ultimate goal, one may hope that strong laser fields could be utilized as tools for preparation, control and detection methods in nuclear physics. Possible applications could be the control of the reaction channels in laser-nucleus interactions, i.e., switch between pair creation, nuclear excitation, fragmentation, fission or other processes. Furthermore, and based on the experience of high-precision laser spectroscopy for atomic and molecular systems, lasers might be employed to measure high-resolution spectra especially of low-lying nuclear states, as well as nuclear properties such as energies, lifetimes, and transition moments. Laser-assisted preparation of nuclear states may also serve to find new effects or reaction channels in nuclear reactions. In addition, some observables may allow to measure properties of nuclei such as transition dipole moments and transition energies independent of nuclear models [35].

From a comparison with atomic physics, it appears obvious that a near-resonant driving of nuclear transitions as studied in [35] is the most promising approach to laser-nucleus interactions. The large transition frequencies in nuclei, however, make this challenging, and require high-frequency laser facilities, possibly assisted by an acceleration of the target nuclei. Such coherent high-frequency light sources, however, are rare as compared to corresponding light sources at optical frequencies. Thus the question arises, whether direct laser-nucleus interactions are also possible and of relevance with super-intense laser fields in the optical frequency region, far off resonance with the considered nuclear transitions. The obvious advantage of this approach is a relaxation of the demands on the facilitated laser source with respect to frequency.

Therefore, in this study, we investigate AC-Stark shifts of single-particle proton states in the presence of off-resonant super-intense laser fields. We find that these shifts may serve as a signature of direct laser-nucleus interactions. In the lab frame, the considered laser fields are in the optical frequency region ( $\mathcal{O}(1 \text{ eV})$ ). Head-on collisions of the laser field with accelerated nuclei allow to increase the frequency and the intensity of the photons in the rest frame of the nuclei. The required nuclear properties are calculated with the help of a relativistic mean-field model. Relativistic mean-field models, and more generally self-consistent mean-field models, provide a wealth of information on the nuclear ground state in a converged calculation, such as the binding energy, the proton, neutron, and charge density, as well as all single-particle wave-functions. The latter are of most relevance for the present study.

We find that with the help of a moderate acceleration of the target nuclei, present and near-future super-intense laser fields may induce AC-Stark shifts which relative to the respective transition frequencies are of similar order as found in typical quantum optical setups. Our primary observable, the AC-Stark shift, is closely related to work in atomic physics in order to facilitate a comparison of these two branches. It should be noted, however, that while the nuclear AC-Stark shift is closely related to

the atomic counterpart, there are some interesting differences, which may allow for physical processes exclusively available in nuclei. These differences will briefly be discussed in the final part. As a first step in this direction, we further study proton root-mean-square (rms) radii and proton densities under influence of off-resonant super-intense laser fields as typical observables in nuclear physics.

The article is structured as follows: In Sec. II, we describe the laser-nucleus interaction employed in this study as well as the nuclear model that is used to calculate the single-particle wave-functions. We discuss the computational procedure and present possible observables. Section III presents the numerical results of the AC-Stark shift calculations and discusses their implications. We also relate them to the atomic case. Section IV discusses and summarizes the results.

## II. THEORETICAL FRAMEWORK

### A. Laser-nucleus interaction

We treat the laser-nucleus interaction in the electric dipole approximation, in which the (non-relativistic) interaction term in the length gauge is given by [3, 37]

$$H_I = -e\vec{E}(t) \cdot \vec{r}. \quad (2.1)$$

Here,  $e = |e|$  is the electron charge,  $\vec{E}(t)$  is the electric field, and  $\vec{r}$  the position operator. For light linearly polarized in  $z$ -direction, this reduces to  $H_I = -eE(t)z$ . The total Hamiltonian of our system is thus

$$H = H_0 + H_I, \quad (2.2)$$

where  $H_0$  denotes the nuclear Hamilton operator that is specified by the nuclear model employed and will be described in Section II C. A few comments on the choice of this interaction are in order. The spatial dependence of the electric laser field is neglected in the dipole approximation due to the small extension of the nucleus of the order of a few Fermi. The magnetic fields can be neglected due to the smallness of the laser-nucleus interaction. Here we have an important difference to atomic systems: Intensities considered large on atomic scales (they compete with the Coulomb force of the nucleus), typically are still weak as compared to the much stronger force between nucleons. Thus a non-relativistic treatment is justified in our case. The nuclear model employed in this study provides a covariant framework for the nuclear ground-state description. Note, however, that the nucleons within the nucleus move non-relativistically. The predominant relativistic feature is the strong spin-orbit force in nuclei, which is an intrinsic ingredient in a covariant description employing strong scalar and vector fields.

In axial symmetry, the proton single-particle wave-functions can be written as [38]

$$\begin{aligned} \psi_i(z, \rho, \phi) &= \psi_i^{\eta\sigma}(z, \rho, \phi) \\ &= \begin{pmatrix} \phi_{\eta_i m_i \pi_i}^{++}(z, \rho) \exp[i(m_i - \frac{1}{2})\phi] \\ \phi_{\eta_i m_i \pi_i}^{+-}(z, \rho) \exp[i(m_i + \frac{1}{2})\phi] \\ \phi_{\eta_i m_i \pi_i}^{-+}(z, \rho) \exp[i(m_i - \frac{1}{2})\phi] \\ \phi_{\eta_i m_i \pi_i}^{--}(z, \rho) \exp[i(m_i + \frac{1}{2})\phi] \end{pmatrix}, \end{aligned} \quad (2.3)$$

where  $n_i, m_i, \pi_i$  are radial quantum number, the projection of the total angular momentum on the symmetry ( $z$ -) axis, and the parity. The corresponding eigenvalue equations for  $m_i$  and  $\pi_i$  read  $\hat{J}_z \psi_i = m_i \psi_i$  and  $\hat{P} \psi_i = \pi_i \psi_i$ . The overlap of two states in axial symmetry is

$$\begin{aligned} \langle \psi_i | \psi_j \rangle &= \delta_{m_i m_j} \delta_{\pi_i \pi_j} 2\pi \int_{-\infty}^{\infty} dz \int_0^{\infty} \rho d\rho \\ &\times \sum_{\eta, \sigma} \phi_{\eta_i m_i \pi_i}^{\eta\sigma}(z, \rho) \phi_{\eta_j m_j \pi_j}^{\eta\sigma}(z, \rho). \end{aligned} \quad (2.4)$$

The condition  $m_i = m_j$  follows from  $\int_0^{2\pi} d\phi e^{i\Delta m \phi} = 0$  for  $\Delta m \neq 0$ . The interaction Hamiltonian  $H_I$  introduces no additional  $\phi$ -dependence and the operator  $z$  commutes with  $J_z$ , hence the condition  $m_i = m_j$  persists. Furthermore, since it is an odd function of  $z$ , we now have for non-vanishing matrix elements the condition  $\pi_i \neq \pi_j$ . The matrix elements read

$$\begin{aligned} \langle \psi_i | z | \psi_j \rangle &= \delta_{m_i m_j} \delta_{\pi_i [\pi_j \times (-1)]} 2\pi \int_{-\infty}^{\infty} dz \int_0^{\infty} \rho d\rho \\ &\times \sum_{\eta, \sigma} \phi_{\eta_i m_i \pi_i}^{\eta\sigma}(z, \rho) \times z \times \phi_{\eta_j m_j \pi_j}^{\eta\sigma}(z, \rho). \end{aligned} \quad (2.5)$$

## B. Observables

We focus on two observables relevant to nuclei exposed to super-intense laser fields. First, the proton energy shifts themselves are—in principle—observable. Second, if the AC Stark shifts are large and the single-particle states are affected to a certain extent, the nuclear density experiences changes, hence density or form-factor related observables become of interest.

### 1. Stark shift

The AC Stark shifts of the proton single-particle states in the laser field can be calculated equivalently to the case of electron states in the atom. A semi-classical calculation of the dynamic Stark shift yields [39, 40]

$$\Delta E_n = \frac{1}{4} \sum_{m, \pm} \frac{\langle n | H_I | m \rangle \langle m | H_I | n \rangle}{\epsilon_n - \epsilon_m \pm \hbar\nu + i\hbar\epsilon}. \quad (2.6)$$

The (unperturbed) single-particle energies are denoted by  $\epsilon_m$ . Note that  $\Delta E$  arises as a second-order perturbation effect since the single-particle wave-functions have good parity (this is also true in the atomic case). As discussed in [39], the quantum-mechanical calculation yields the same result in the limit of large photon number, which applies to our study.

In the limit  $\hbar\nu \ll \Delta\epsilon = \epsilon_n - \epsilon_m$ , i.e., for laser field photon energies well below the nuclear transition frequencies, the laser-frequency dependence in the denominator drops out, leaving us with

$$\Delta E_n^{\ll} = \frac{1}{2} \sum_{m \neq n} \frac{\langle n | H_I | m \rangle \langle m | H_I | n \rangle}{\epsilon_n - \epsilon_m}. \quad (2.7)$$

The same expression is obtained in a time-averaged calculation in the adiabatic limit [41]. In the following, we use expression Eq. (2.7) to quantify the Stark shifts, since we focus on the off-resonant excitation of the nuclear transitions, such that  $\hbar\nu \ll \Delta\epsilon$  is fulfilled in all cases considered.

### 2. Density-related observables

The actual proton density of the nucleus exposed to the laser field can be computed by taking into account perturbatively the corrections to the wave-functions due to the interaction with the laser field through  $H_I$ . We write the spatial part  $|\phi_n\rangle$  of the total wave function in second-order perturbation theory as

$$|\phi_n\rangle = |n^0\rangle + |n^1\rangle + |n^2\rangle, \quad (2.8)$$

where the superscript indicates the order of perturbation. In the adiabatic limit, one obtains [41]

$$\begin{aligned} |\phi_n\rangle &= |n^0\rangle + \sum_k a_{kn}^1 \sin(\omega_L t) |k\rangle \\ &+ \sum_k a_{kn}^2 \sin^2(\omega_L t) |k\rangle. \end{aligned} \quad (2.9)$$

The first ( $a_{kn}^1$ ) and second ( $a_{kn}^2$ ) order expansion coefficients read [41]

$$a_{nn}^1 = \langle n^0 | n^1 \rangle = 0, \quad (2.10a)$$

$$a_{kn}^1 = \langle k^0 | n^1 \rangle = \frac{\langle k | H_I | n \rangle}{E_n^0 - E_k^0} \quad (k \neq n), \quad (2.10b)$$

$$a_{nn}^2 = \langle n^0 | n^2 \rangle = -\frac{1}{2} \sum_{m \neq n} \frac{|\langle m | H_I | n \rangle|^2}{(E_n^0 - E_m^0)^2}, \quad (2.10c)$$

$$\begin{aligned} a_{kn}^2 = \langle k^0 | n^2 \rangle &= \sum_{m \neq n} \frac{\langle k | H_I | m \rangle \langle m | H_I | n \rangle}{(E_n^0 - E_k^0)(E_n^0 - E_m^0)} \\ &- \underbrace{\frac{\langle k | H_I | n \rangle \langle n | H_I | n \rangle}{(E_n^0 - E_k^0)^2}}_{=0} \quad (k \neq n). \end{aligned} \quad (2.10d)$$

The last addend of Eq. (2.10d) vanishes from parity. In this study we are interested in the time-averaged single-particle densities from which we can compute the proton rms radius and the proton quadrupole moment. Using

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt \sin(ct) = 0, \quad (2.11a)$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt \sin^2(ct) = \frac{1}{2}, \quad (2.11b)$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt \sin^3(ct) = 0, \quad (2.11c)$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt \sin^4(ct) = 3/8, \quad (2.11d)$$

we can compute the average single-particle density in coordinate space as [42]

$$\bar{\rho}_l = \overline{\phi_l^* \phi_l} \neq \overline{\phi_l^*} \times \overline{\phi_l}, \quad (2.12)$$

and obtain

$$\begin{aligned} \bar{\rho}_l = & \phi_l^{0*} \phi_l^0 + \sum_i a_{li}^2 \phi_l^{0*} \phi_i \\ & + \sum_{i,j} \left( \frac{3}{8} a_{li}^2 a_{lj}^2 + \frac{1}{2} a_{li}^1 a_{lj}^1 \right) \phi_i^{0*} \phi_j^0. \end{aligned} \quad (2.13)$$

This density is used for the calculation of the ground-state proton radius and deformation as shown below. These are standard observables used to calibrate and judge the predictive power of nuclear models using known experimental data [43].

The rms radius of the proton density is defined as [42]

$$r_{rms}^p = \sqrt{\frac{\int d^3x r^2 \rho^p(\vec{x})}{\int d^3x \rho^p(\vec{x})}}, \quad (2.14)$$

with  $r = \sqrt{x^2 + y^2 + z^2}$ ,  $\rho^p(\vec{x})$  is the proton point density. Note that this definition also holds for non-spherical density distributions. The rms radius is related to the spatial extension of the density distribution. The experimentally accessible quantity in nuclei is the nuclear charge radius which can be extracted from the corresponding measured form factor.

The spherical quadrupole moment in axial symmetry reads [42]

$$Q_{20} = \frac{1}{2} \sqrt{\frac{5}{4\pi}} \int d^3x \rho^p(\vec{x}) (2z^2 - r^2), \quad (2.15)$$

where  $r = \sqrt{x^2 + y^2}$ . Positive values of  $Q_{20}$  denote cigar-like shapes, while negative values correspond to disk-like nuclear density distributions. Since the quadrupole moment integrates over the proton density it shows a mass dependence. A dimensionless quantity without such mass dependence is given by

$$\beta_2 \equiv \beta_{20} = \frac{4\pi}{3nR^2} Q_{20}, \quad (2.16)$$

where  $n = \int d^3x \rho^p$ , and  $R = 1.2 \text{ fm} \times A^{1/3}$  is an approximation of the nuclear radius ( $A$  is the total number of nucleons).

### C. The nuclear model

A quantitative estimate of the nuclear dynamic Stark shifts demands realistic proton single-particle wave functions which we obtain by employing the relativistic mean-field (RMF) model for the ground-state calculation of the nucleus. Though we have no guaranty that these wave functions yield a close approximation to nature, the success of the RMF approach supports our choice [44, 45]. Moreover, these wave functions do not suffer from known deficiencies of other approaches, e.g., the wrong asymptotics of wave functions obtained in a harmonic oscillator potential.

The RMF model [44, 46, 47, 48] has historically been designed as a renormalizable meson-field theory for nuclear matter and finite nuclei. The realization of nonlinear self-interactions of the scalar meson led to a quantitative description of nuclear ground states. As a self-consistent mean-field model (for a comprehensive review see [46]), its ansatz is a Lagrangian or Hamiltonian that incorporates the effective, in-medium nucleon-nucleon interaction. In contrast to macroscopic-microscopic approaches, no assumptions on the nuclear potential or density are made. RMF models yield the binding energy and all single-particle wave-functions in one calculation, from which several other kinds of observables can be obtained.

Recently, self-consistent models have undergone a reinterpretation [45] which explains their quantitative success in view of the facts that nucleons are composite objects and that the mesons employed in RMF have only a loose correspondence to the physical meson spectrum [49]. They are seen as covariant *Kohn-Sham schemes* [53] and as approximations to the true functional of the nuclear ground state. According to the *Hohenberg-Kohn theorem*, the exact ground-state functional does exist. However, this theorem does not provide a handle to construct it (it is non-constructive). As a Kohn-Sham scheme, the RMF model can incorporate certain ground-state correlations and yields a ground-state description beyond the *literal* mean-field picture. RMF models are effective field theories for nuclei below an energy scale of  $\Lambda \approx 1 \text{ GeV}$ , separating the long- and intermediate-range nuclear physics from short-distance physics, involving, i.e., short-range correlations, nucleon form factors, vacuum polarization etc, which is absorbed into the various terms and coupling constants.

The strong attractive scalar ( $S \approx -400 \text{ MeV}$ ) and repulsive vector ( $V \approx +350 \text{ MeV}$ ) fields provide both the binding mechanism ( $S + V \approx -50 \text{ MeV}$ ) and the strong spin-orbit force ( $S - V \approx -750 \text{ MeV}$ ) of both right sign and magnitude.

The RMF model is based on phenomenology and needs an adjustment of its (phenomenologically introduced)

coupling constants [43]. We have chosen the parameterization NL3 [54], which is among the most successful parameterizations available.

#### D. Computational procedure

The stationary mean-field equations are solved with a C++ code on a grid in coordinate space in axial symmetry. The wave functions are written out and then processed to compute the dipole matrix elements  $\langle a|z|b\rangle$  between respective proton states  $a$  and  $b$ . Matrix manipulations and integrations are done using Python together with the modules NumArray [55] and SciPy [56]. We have neglected pairing in our mean-field calculation to be consistent with the following computation of the matrix elements and the Stark shifts. While pairing is important for a highly accurate description of ground-state energy and deformation, it is not relevant for our purpose.

Uncertainties in the calculations of the dipole matrix elements stem from the calculated radial components of the wave functions. Still, mean-field wave-functions can be considered realistic for nuclei close to the valley of stability and for well-bound states, which we consider here. Thus we will not reach the accuracy reached in QED calculations, but we still can expect solid quantitative predictions of the AC Stark shifts. This justifies further approximations that introduce uncertainties within this framework. We have neglected the influence of the center of mass motion of the nucleus and the (very weak) coupling of the neutrons to the laser, thus no effective charges were introduced in our calculations. Furthermore, all effects beyond the electric dipole approximation, which are related to the magnetic field contributions of the laser field have been omitted. In this respect, one should note that it is not the laser field intensity itself, but rather the effective coupling  $H_I$  relative to the mean-field Hamiltonian  $H_0$  which determines whether the electromagnetic field induces large perturbation on the nuclear ground state or not. Thus approximations are valid in the nuclear case for intensities where the same approximations break down for the calculations of atomic or molecular systems.

In this work we assume that a spheroidal nucleus will have its symmetry axis aligned with the direction of the laser field. This way, the interaction with the laser field does not destroy axial symmetry of the system which we employ in our numerical solution of the mean-field equations. Since most spheroidal nuclei possess no static dipole moment, this alignment will not take place naturally. However, alignment can take place under the following conditions: a) the nucleus has a reflection-asymmetric ground-state shape and thus a static electric dipole moment (there are rare cases) [57]; b) we employ an additional electric field gradient in the polarization direction of the laser field. The interaction with the quadrupole moment of the nucleus then leads to alignment since the interaction energy of the quadrupole mo-

ment with the external static electric field is given by  $W = -\frac{1}{4}\left(\frac{\partial E_z}{\partial z}\right)eQ_z$  assuming that the electric field is pointing in  $z$  direction. Without such an alignment, the nucleus exposed to the laser field will experience shape changes that lead to triaxial shapes and the dipole matrix elements will slightly differ. The size of the Stark shifts, however, will be similar to the ones calculated in the aligned case.

#### E. Laser-Nucleus collisions

Both laser frequency and intensity in the nuclear rest frame can be effectively increased by letting the nucleus and laser collide head-on. In the rest frame of the nucleus, the Doppler shifted electric field strength  $E_N$  and the frequency  $\nu_N$  are given by

$$E_N = \sqrt{\frac{1+\beta}{1-\beta}} E_L = (1+\beta)\gamma E_L, \quad (2.17)$$

$$\nu_N = \sqrt{\frac{1+\beta}{1-\beta}} \nu_L = (1+\beta)\gamma \nu_L, \quad (2.18)$$

where subscript  $N$  denotes the nuclear rest frame and  $L$  the laboratory frame, respectively.

For  $\gamma$  factors of about 1000 (4000), one has  $\beta \approx 1$  (1), and we obtain  $(1+\beta)\gamma \approx 2000$  (8000). Since the laser intensity is proportional to  $E^2$ , we find amplification by a factor of  $4.0 \times 10^6$  ( $6.4 \times 10^7$ ). Assuming lab-frame intensities of  $I_L \approx 10^{22-24}$  W/cm<sup>2</sup>, we can reach  $I_N \approx 10^{28-31}$  W/cm<sup>2</sup>. Intensities of  $I_L \approx 10^{28}$  W/cm<sup>2</sup> are in reach in the near future, and the higher the laser intensity, the smaller is the necessary  $\gamma$  factor of the accelerated nuclei. As will be shown below, in order to reach AC Stark shifts comparable to typical shifts in atomic systems with respect to the transition frequencies, only moderate  $\gamma$  shifts are necessary. For these cases of  $\gamma \approx 1000-4000$ , employing optical lasers with  $E \approx 1$  eV, photon energies of about 2–10 keV result, which is still smaller than typical energy differences of proton single-particles energies of a few MeV (deeply bound states) or some hundreds of keV (near the Fermi edge). Choosing IR-lasers in the first place yields even smaller photon energies.

In the following, we discuss frequencies and intensities in the nuclear rest frame. Note that it is not important whether the assumed values are reached via a large velocity as compared to the lab frame, a powerful laser facility, or a combination of both. Both high-intensity laser as well as ion accelerators are available today or in the near future, albeit mostly in separate places. A promising ansatz to reach experimental conditions as required for the physics discussed here would be to install and combine both types of facilities in one laboratory.

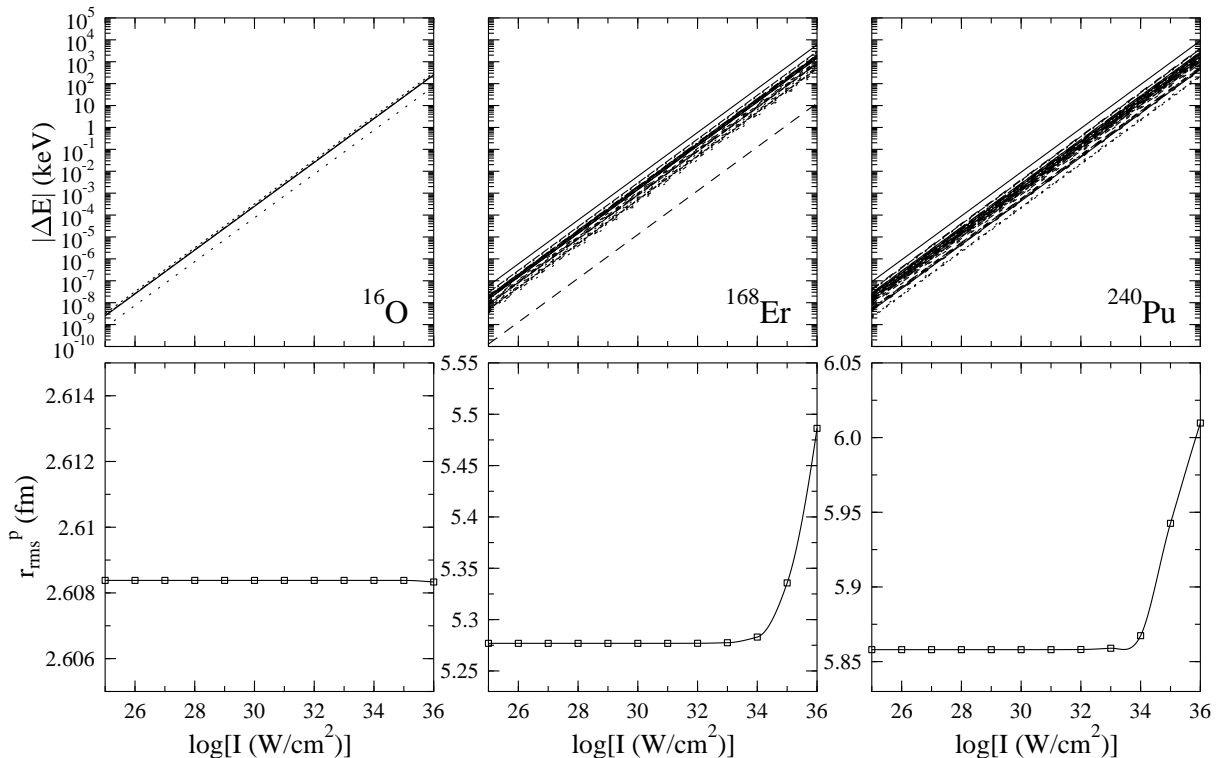


FIG. 1: AC-Stark shifts of the proton single-particle states in the laser field (upper row) as a function of laser intensity in the nuclear rest frame for the nuclei as indicated, and proton rms radii (lower row) as a function of laser intensity in the nuclear rest frame. Each line in the upper figures corresponds to a Stark shift of a proton single-particle level. The widths of these bands characterize the spread in these shifts. In the lower figure, the square dots indicate the calculated results, which for convenience are connected by the thin line.

### III. RESULTS

Figure 1 shows the AC Stark shifts of single proton states in the nuclei  $^{16}\text{O}$ ,  $^{168}\text{Er}$ , and  $^{240}\text{Pu}$ , as well as the corresponding rms proton radii. We have chosen these nuclei as typical representatives of light, intermediate, and heavy nuclei. Their lowest measured E1 excitations lie at 7.117 MeV ( $^{16}\text{O}$ ), 1.359 MeV ( $^{168}\text{Er}$ ), and 0.555 MeV ( $^{240}\text{Pu}$ ), respectively [58]. Thus, transitions will not be excited by the considered laser energies of  $\mathcal{O}(\text{keV})$  in the nuclear rest frame. Lower excitations of even parity would require two- or higher-order photon processes, and their energies are still more than 20 keV above the ground state energy. Hence we can treat the Stark effect separately from nuclear excitation mechanisms.

Shifts of  $\approx 1$  keV are reached at intensities of roughly  $10^{34} \text{ W/cm}^2$  for oxygen, and  $10^{32} \text{ W/cm}^2$  for the heavier systems. As discussed above, these shifts are approximately a factor of 10-1000 smaller than typical energy differences of single-particle levels close to the Fermi edge. As expected, in absolute terms, they are much larger than shifts appearing in atomic systems, but may also surpass them in relative terms, see the end of this section for details. The size of the shifts depends both on

the matrix elements  $\langle m | H_I | n \rangle$  as well as on the number of states contributing with dipole moments and their corresponding single-particle energies. Since oxygen has only 8 protons, the effects are rather small. There are no significant changes in the proton rms radius. The Stark shifts in  $^{168}\text{Er}$ , and  $^{240}\text{Pu}$  are (on the average) larger than in oxygen due to the increased number of states. Also, changes in the proton radii set in above  $I = 10^{33} \text{ W/cm}^2$ .

For the medium and heavy nuclei under consideration, shape changes of the nucleus (as reflected in the increasing rms radii) also lead to an increase of the quadrupole moment of these systems, see Section II B 2, changing the moments of inertia. This will in turn alter the rotational excited states of these systems.

Figure 2 displays the proton density for laser intensities of  $I = 10^{25} \text{ W/cm}^2$  (left) and  $I = 10^{35} \text{ W/cm}^2$  (right). At  $10^{25} \text{ W/cm}^2$ , the density profile resembles the ground-state density, no differences are visible. This is due to the fact that Stark shifts and, correspondingly, changes of the single-particle wave-functions are small. At the higher intensity, Stark shifts reach values of a few hundred keV. This is certainly the limit of our adiabaticity assumption. The proton density is slightly extended in  $z$ - and  $r$ -directions. Even more prominently, the den-

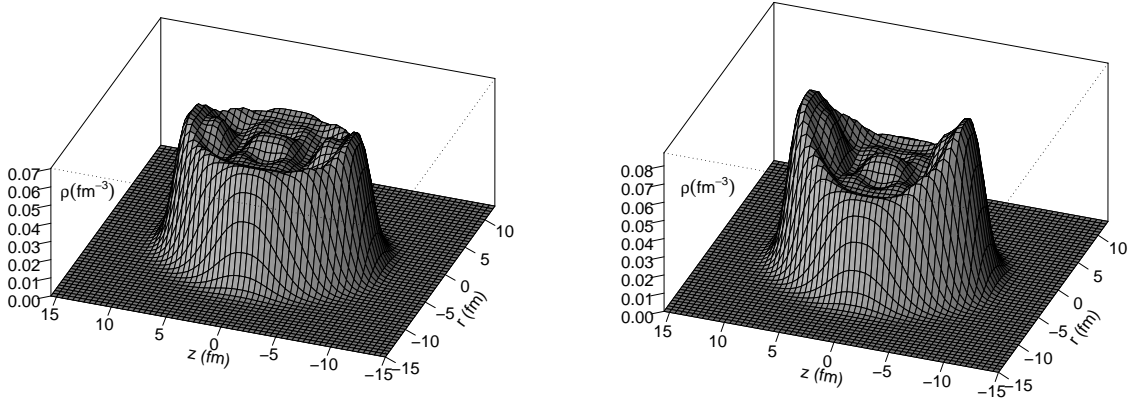


FIG. 2: Proton density of  $^{240}\text{Pu}$  for intensities of  $I = 10^{25} \text{ W/cm}^2$  (left) and  $I = 10^{35} \text{ W/cm}^2$  (right).

sity close to the center of the nucleus is reduced, and the *poles* of the proton density get enhanced. This might be related to the fact that the dipole matrix elements yield largest contributions for states close to the Fermi edge, where high total angular momentum projections on the  $z$ -axis occur. These states are localized at large  $z$ -values and thus lead to the visible enhancement. This rearrangement of the nucleus leads to an increase of the quadrupole moment, see Eq. (2.15). Significant changes in this observable correspond to the respective changes in the rms radii that are shown in Fig. 1.

We would like to classify the various processes taking place for a nucleus in a super-intense laser-field according to the nuclear rest-frame laser intensity (see Ref. [7, 8, 9, 59] for a discussion of effects relevant to laser-nucleus physics). The following hierarchy of effects can be constructed, going from low to high laser intensities in the nuclear rest frame:

- $I < 10^{29} \text{ W/cm}^2$ : a) radiation from scattering off of the nucleus in the laser field; b) radiation from electrons surrounding the nucleus if the ion is not fully stripped; c) the AC Stark shifts are already comparable to the typical atomic shifts in relation to the transitional energies of single-particle states

- $I \approx 10^{29} \text{ W/cm}^2$ : This is the critical field strength [60, 61, 62] at which  $e^+e^-$  pair creation sets in [33, 34, 63], additionally radiation is generated by created electrons or positrons that oscillate for a few cycles within the laser field

- $I \geq 10^{32} \text{ W/cm}^2$ : Direct laser-nuclear interactions become non-negligible, AC Stark shifts of proton states lead to a slight structural change of the nucleus, weak quadrupole oscillations take place in the laser field; very weak quadrupole radiation sets in.

We can compare the nuclear AC Stark effects with similar situations for atomic systems. In typical non-resonant laser-atom systems which aim at measuring the AC Stark shifts in moderate laser fields, the relation of the energy shifts due to the laser fields compared to typical energy differences of  $\mathcal{O}(\text{eV})$  is  $\approx 10^{-12} - 10^{-10}$  [64, 65]. In the nuclear case with energy differences of  $\mathcal{O}(\text{MeV})$ , this would correspond to AC Stark shifts of the order of  $10^{-9} - 10^{-7} \text{ keV}$ , as found in the low intensity regime of Fig. 1. The corresponding intensities of  $I = 10^{25} - 10^{27} \text{ W/cm}^2$  are close to intensities that can be presently reached or are envisaged in the near future. Thus, nuclear AC Stark shifts that are similarly related to the typical transition energies as in the atomic case can in the future be expected even without a pre-acceleration of the nuclei. It remains to be seen if these Stark shifts can be directly measured. Such kind of measurements, however, would demonstrate the direct laser-nucleus interactions in a very concise way. We would like to point out that the framework of our Stark-shift calculations is based on continuous wave lasers, while in most realistic situations (laser-nucleus collisions, or super-intense lasers incident on a fixed target) laser pulses will be employed. There, the calculated Stark shift sizes correspond to the central region of the laser pulse.

In addition to the structural changes, due to the incident field, the nucleus will also experience an oscillating center-of-mass motion, resulting in dipole radiation perpendicular to the direction of the laser electric field and the beam axis (we do not consider the drift motion in beam direction, consistent with the non-relativistic treatment of the light-matter interaction as discussed in Secs. II A and II D). The oscillation extent  $\Delta x_{\text{pol}}$ , which is twice the amplitude of oscillation in polarization di-

rection, can be calculated classically for a structureless particle of charge  $q$  [66] in an electromagnetic field, yielding  $\Delta x_{pol} = (2|q|E_N)/(m\omega_N^2)$ . We estimate  $\Delta x_{pol}$  for the nucleus  $^{168}\text{Er}$ , using  $|q| = 68 e$ ,  $m \approx 168 u$ , for  $E_N = 1.0 \times 10^{16} \text{ V/cm}$ . As photon energies, we choose  $\hbar\omega = 1 \text{ eV}$  and  $\hbar\omega = 1 \text{ keV}$ , respectively. The resulting oscillation extent is  $\Delta x_{pol} \approx 3 \cdot 10^{10} \text{ fm}$  for  $\hbar\omega = 1 \text{ eV}$  and  $\Delta x_{pol} \approx 3 \cdot 10^4 \text{ fm}$  for  $\hbar\omega = 1 \text{ keV}$ . This should be compared to the size of the nucleus, which is on the order of 6 fm. Note also that here the nucleus is assumed to be free of electrons, which increases the charge-to-mass ratio as compared to the case of a singly charged ionic core after a single electron ionization. This enhances the response of the nucleus to the incident field. The dipole-type nuclear center of mass motion yields radiation, with total radiation power given classically by [67]

$$P = \frac{c^2 Z_0 k^4}{12\pi} |\vec{p}|^2, \quad (3.1)$$

where  $Z_0 = \sqrt{\mu_0/\epsilon_0}$  is the impedance of the vacuum, and  $\vec{p} = \int \vec{r} \rho^p(\vec{r}) d^3r$  is the dipole moment, respectively. Assuming this radiation to be emitted by photons of energy  $E = \hbar\omega$ , we can semi-classically estimate the time needed for the emission of one photon by

$$P = \frac{W}{t} = \frac{\hbar\omega}{t} \Leftrightarrow t = \frac{\hbar\omega}{P}. \quad (3.2)$$

If we equate  $\Delta x_{pol}$  with the length entering the dipole moment, for  $^{168}\text{Er}$  we obtain  $|\vec{p}| = 68 e \times \Delta x_{pol}$ . For photon energies of  $\hbar\omega = 1 \text{ eV}$  we estimate  $t_N = 2 \times 10^{-21} \text{ s}$ , for  $\hbar\omega = 1 \text{ keV}$  we obtain  $t_N = 2 \times 10^{-18} \text{ s}$ .

These emission times have to be compared to typical laser pulse durations of 1-100 fs, which is the duration over which the required field strength can be maintained. The average amount of signal photons then further depends on the repetition rate of the experimental setup. The radiation generated from nuclear quadrupole shape oscillations is suppressed as compared to this dipole radiation due to a center-of-mass motion, which is unfortunate, since quadrupole radiation is a unique signal for the above discussed structural changes of the nucleus. Its detection may become feasible, however, once the required laser intensities in the laboratory frame become available with high repetition rate. Also, by preparing large ensembles of nuclei flying head-on into the laser beam, the number of individual interactions may become large enough for the detection of quadrupole radiation.

Finally, we would like to return back to the structural properties of nuclei in the laser field in contrast to atomic systems. The neutrons in nuclei are likely to adiabatically follow the periodic changes of the proton states. Thus, not only the proton density, but also the mass density undergoes (tiny) quadrupole oscillations. Furthermore, non-closed-shell nuclei are superfluid systems, where the short-range pairing correlations soften the Fermi edge and allow angular momentum paired nucleon-pairs to scatter into energetically higher-lying orbits. The presence of the laser field will also effect the

continuum states in the nucleus and thus we may expect that the pairing correlations will be altered. This then, in turn, affects the moment of inertia for deformed nuclei.

#### IV. CONCLUSIONS AND OUTLOOK

Our work is motivated by the hope that externally controllable super-intense electromagnetic fields could enhance preparation, control and detection methods in nuclear physics similar to the tremendous success of such control methods in atomic and molecular physics. We have shown that a combination of cutting-edge lasers and ion accelerators available today or in the near future opens a pathway for the study of direct laser-nuclear interactions. These interactions do not involve intermediate particles such as external electrons accelerated by the laser pulse. The potential for such applications obviously will increase with improving laser and accelerator facilities. Ultimately, resonant laser-nucleus interactions can be expected to be the most promising candidate for a direct manipulation of nuclei by coherent light, but these require high-frequency laser sources. As discussed in this article, however, also off-resonant super-intense laser fields in the optical frequency region may induce signatures of direct laser-nuclei interactions. An important task, of course, remains to find convenient observables or experimental setups where the direct laser-nucleus interactions can be observed or even crucially influence the outcome of a desired measurement. Examples could be processes sensitive to resonance conditions subject to laser-induced shifts, or the observation of weak quadrupole radiation emitted due to oscillatory excitations of the nuclei.

Our main observable, the AC-Stark shift, is closely related to work in atomic physics in order to facilitate a comparison of these two branches. Despite the similarities of the nuclear AC-Stark shift calculations to the atomic case, however, several differences between the atomic and the nuclear case should be noted. First, in nuclei, the electromagnetic force is not the strongest force present. The electric field generated by the laser field is a perturbation on top of the inter-proton Coulomb force, being itself a perturbation on the strong nucleon-nucleon force governing to a major extent the structure of the nucleus. In atoms, however, the electromagnetic force between electrons and the nucleus (and between electrons and electrons) is governing structure of the atom. Second, nuclei do not possess a central Coulomb potential. Furthermore, the Coulomb force between protons within the nucleus is much stronger than the inter-atomic Coulomb force between electrons and the nucleus because of the much smaller size of the nucleus. Third, nuclei possess rich structural properties, i.e., collective excitations (rotations, vibrations, giant resonances), as well as single-particle excitations. Especially with respect to collectivity, they resemble more molecules than atoms. Obviously, these differences are of especial interest since



they reflect the unique properties of nuclei as compared to atomic systems. Thus for the future, an investigation focused on typical observables in nuclear physics, possibly based on the specific differences of atomic and nuclear physics, is desirable. The proton rms radii discussed in this work are a first step in this direction, even though we found that a measurement of laser-induced changes in these radii is rather challenging.

Finally, our study could also be of relevance for situations where *indirect* laser-nucleus-interactions occur, as it provides data on the modification of nuclear properties under the influence of external fields. For example, a laser field utilized to create secondary particles which in turn interact with nuclei can also be expected to modify the interaction of the nuclei with the auxiliary particles.

In summary, we have studied the direct off-resonant interaction of super-intense fields in the optical frequency

region with nuclei. In particular, laser fields which are in the optical region in the lab frame were considered, possibly in combination with an acceleration of the target nuclei. Then already field intensities available now or in the near future can induce AC-Stark shifts of the same order as in typical quantum optical systems relative to the respective transition frequencies. We thus expect these direct laser-nucleus interaction to become of relevance together with other super-intense light-matter interaction processes such as pair creation.

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